Coupled Circuit-Interconnect Analysis using Stable Arnoldi-based Model Order Reduction

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The dense three-dimensional packaging used in compact electronic systems often produce magnetic interactions which interfere with system performance. Such effects are difficult to simulate because they occur only as a result of an interaction between the field distribution in a complicated geometry of conductors, and the circuitry connected to those conductors. For structures small compared to a wavelength, electromagnetic interactions between conductors can be represented arbitrarily accurately using a densely coupled resistor, inductor, and capacitor (RLC) network [1].

A standard way to improve the efficiency of coupled circuit-interconnect simulation is to use Padé-based reduced order models, but the resulting Padé approximates can still be unstable even when generated from stable RLC circuits. It has been shown that, for certain classes of RC circuits, congruence transforms, like the Arnoldi algorithm, can generate guaranteed stable and passive reduced-order models [2]. A computationally efficient model-order reduction technique, the coordinate-transformed Arnoldi algorithm, can be used to generate arbitrarily accurate and guaranteed stable reduced-order models for general RLC circuits [3].

The system of equations resulting from modified nodal analysis on an RLC network consisting of M inductors and N nodes can be written as $\boldsymbol{A}\,\dot{\boldsymbol{x}} = \boldsymbol{x} + \boldsymbol{b}u$, $y = \boldsymbol{c}^T\boldsymbol{x}$. where $\boldsymbol{A} = -\boldsymbol{\mathcal{R}}^{-1}\boldsymbol{\mathcal{L}} \in \mathbb{R}^{n\times n}$, $\boldsymbol{\mathcal{L}}$ is the matrix of capacitances and inductances, $\boldsymbol{\mathcal{R}}$ contains the conductance matrix and incidence matrices associated with the inductor currents, u is the vector of source currents, y is the vector of output voltages, and $n = N + M$. It can be shown that the Arnoldi algorithm will produce a stable reduced order model if $\boldsymbol{A}$ is negative semidefinite. Although the matrices $\boldsymbol{\mathcal{L}}$ and $\boldsymbol{\mathcal{R}}$, generated by modified nodal analysis of an RLC circuit with positive elements, are in general positive semidefinite, the matrix $\boldsymbol{A} = -\boldsymbol{\mathcal{R}}^{-1}\boldsymbol{\mathcal{L}}$ is not necessarily negative semidefinite. Consider a change of variable, $\tilde{\boldsymbol{x}} = \boldsymbol{\mathcal{L}}^{\frac{1}{2}}\boldsymbol{x}$, where $\boldsymbol{\mathcal{L}}^{\frac{1}{2}}$ is the unique symmetric, positive definite square root of the symmetric, positive definite matrix $\boldsymbol{\mathcal{L}}$. The modified system matrix is now given by $\tilde{\boldsymbol{A}} = -\boldsymbol{\mathcal{L}}^{\frac{1}{2}}\boldsymbol{\mathcal{R}}^{-1}\boldsymbol{\mathcal{L}}^{\frac{1}{2}}$ which is negative semidefinite. The actual computation of $\boldsymbol{\mathcal{L}}^{\frac{1}{2}}$ can be avoided by using an $\boldsymbol{\mathcal{L}}$-orthogonal version of the Arnoldi algorithm, which when applied to $\tilde{\boldsymbol{A}}$ is guaranteed to produce a stable reduced order model.

[1] Albert E. Ruehli. Equivalent Circuit Models for Three-Dimensional Multiconductor Systems. *IEEE Transactions on Microwave Theory and Techniques*, MTT-22(3):216–221, March 1974.

[2] K. J. Kerns, I. L. Wemple, and A. T. Yang. Stable and efficient reduction of substrate model networks using congruence transforms. In *IEEE/ACM International Conference on Computer Aided Design*, pages 207 – 214, San Jose, CA, November 1995.

[3] L. Miguel Silveira, Mattan Kamon, Ibrahim Elfadel, and Jacob K. White. A coordinate-transformed Arnoldi algorithm for generating guaranteed stable reduced-order models of rlc circuits. In *International Conference on Computer Aided-Design*, pages 288–294, San Jose, California, November 1996.