Direct Computation of Reduced-Order Models for Circuit Simulation of 3-D Interconnect Structures*

L. Miguel Silveira Mattan Kamon Jacob White

Research Laboratory of Electronics
Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
Cambridge, MA 02139

Tel: (617)-253-7307 Fax: (617)-258-7864

e-mail: {lms,matt,white@rle-vlsi.mit.edu}

1 Introduction

The dense three-dimensional packaging now commonly used in compact electronic systems may produce magnetic interactions which interfere with system performance. Such effects are difficult to simulate because they occur only as a result of an interaction between the field distribution in a complicated geometry of conductors, and the circuitry connected to those conductors. One approach to performing the coupled simulation is to use a 3-D field solver to compute the frequency-dependent impedance matrix associated with the geometry of conductors, and then use frequency-domain fitting and model-order reduction to construct a model suitable for circuit simulation [1].

In order to use frequency-domain fitting as described above, it is necessary to use the field solver to compute impedance matrices at dozens of frequency points, and this is computationally expensive. It is possible to derive a more efficient approach by exploiting the fact that 3-D field solvers typically use Krylov-subspace based iterative methods. These iterative methods can provide more than just a solution at a particular frequency, they can be used to directly construct reduced-order models [2].

In this paper, we present a numerically robust and accurate approach for computing reduced-order models of complicated 3-D structures. The approach is based on using the multipole-accelerated program FASTHENRY [3], combined with the GMRES Krylov-subspace algorithm [4]. We begin, in section 2, by describing the mesh-formulation approach of FASTHENRY. In section 3, the standard Padé approximation approach as well as a GMRES-based approach are derived. In section 4 results are presented comparing the accuracy of the two model-order reduction methods on a package example. Finally, in section 5, we present conclusions and acknowledgments.

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2 The Mesh Formulation Approach

The frequency dependent resistance and inductance matrices describing the terminal behavior of a set of conductors can be rapidly computed with the multipole-accelerated mesh-formulation approach as implemented in FASTHENRY [3]. To describe the approach, consider that each conductor is approximated as piecewise-straight sections. The volume of each straight section is then discretized into a collection of parallel thin filaments through which current is assumed to flow uniformly.

To derive a system of equations for the filament currents, we start by assuming the system is in sinusoidal steady-state and following the partial inductance approach in [5], the branch current phasors can be related to branch voltage phasors by

$$\boldsymbol{V}_b = (\boldsymbol{R} + \boldsymbol{j}\omega\boldsymbol{L})\boldsymbol{I}_b = \boldsymbol{Z}\boldsymbol{I}_b \tag{1}$$

where $\boldsymbol{V}_b$, $\boldsymbol{I}_b \in \mathbb{C}^b$, b is the number of branches (number of current filaments), and ω is excitation frequency. The entries of the diagonal matrix $\boldsymbol{R} \in \mathbb{R}^{b\times b}$ represent the DC resistance of each current filament, and $\boldsymbol{L} \in \mathbb{R}^{b\times b}$ is the dense matrix of partial inductances.

Kirchhoff's voltage law, which implies that the sum of branch voltages around each mesh (a mesh is any loop of branches in the graph which does not enclose any other branches) in the network is represented by

$$\boldsymbol{M}\boldsymbol{V}_b = \boldsymbol{V}_s \qquad \boldsymbol{M}^T\boldsymbol{I}_m = \boldsymbol{I}_b, \tag{2}$$

where $\boldsymbol{V}_s \in \mathbb{C}^m$ is the mostly zero vector of source branch voltages, $\boldsymbol{I}_m \in \mathbb{C}^m$ is the vector of mesh currents, $\boldsymbol{M} \in \mathbb{R}^{m\times b}$ is the mesh matrix. Combining (2) and (1) yields

$$\boldsymbol{M}\boldsymbol{Z}\boldsymbol{M}^T\boldsymbol{I}_m = \boldsymbol{V}_s. \tag{3}$$

The complex admittance matrix which describes the external terminal behavior of the conductor system, denoted $\boldsymbol{Y}_r = \boldsymbol{Z}_r^{-1}$, can by derived from (3) by noting that

$$\tilde{\boldsymbol{I}}_s = \boldsymbol{Y}_r\tilde{\boldsymbol{V}}_s, \tag{4}$$

where $\tilde{\boldsymbol{I}}_s$ and $\tilde{\boldsymbol{V}}_s$ are the vectors of terminal source currents and voltages. Therefore, to compute the i^{th} column of $\boldsymbol{Y}_r$, solve (3) with a $\boldsymbol{V}_s$ whose only nonzero entry corresponds to $\tilde{\boldsymbol{V}}_{s_i}$, and then extract the entries of $\boldsymbol{I}_m$ associated with the source branches.

To solve (3) by Gaussian Elimination would require $\mathcal{O}(m^3)$ operations. To improve the situation, FastHenry uses a multipole-accelerated preconditioned GMRES iterative algorithm [4], which requires $\mathcal{O}(b)$ operations. This follows from the fact that at each iteration of GMRES the dense matrix-vector product, $(\boldsymbol{M}\boldsymbol{Z}\boldsymbol{M}^T)\boldsymbol{I}_m$, can be approximated in $\mathcal{O}(b)$ operations using a hierarchical multipole algorithm [6].

The above process produces an admittance matrix for a single frequency, ω. Thus, for *each* frequency point desired, $\boldsymbol{Z}$ must be recomputed, and (3) solved for each column.

3 Reduced-Order Modeling

In order to derive an efficient algorithm for computing reduced-order models, consider the state-space representation of (3). To that end, expand $\boldsymbol{Z}$ into $\boldsymbol{R} + s\boldsymbol{L}$ and for a given entry, $\boldsymbol{Y}_{ij}$, of the admittance matrix $\boldsymbol{Y}_r$, choose $\boldsymbol{d}$ such that $\boldsymbol{V}_s = \boldsymbol{d}\tilde{\boldsymbol{V}}_{s_j}$, and $\boldsymbol{c}$ such that $\tilde{\boldsymbol{I}}_{s_i} = \boldsymbol{c}^T\boldsymbol{I}_m$. This gives

$$\begin{aligned} s(\boldsymbol{M}\boldsymbol{L}\boldsymbol{M}^T)\boldsymbol{I}_m &= -(\boldsymbol{M}\boldsymbol{R}\boldsymbol{M}^T)\boldsymbol{I}_m + \boldsymbol{d}\tilde{\boldsymbol{V}}_{s_j} \\ \tilde{\boldsymbol{I}}_{s_i} &= \boldsymbol{c}^T\boldsymbol{I}_m \end{aligned} \tag{5}$$

With this representation, the transfer function for the given entry can be written as

$$\frac{\tilde{\boldsymbol{I}}_{s_i}}{\tilde{\boldsymbol{V}}_{s_j}} = \boldsymbol{Y}_{ij}(s) = \boldsymbol{c}^T\left(\boldsymbol{I} - s\boldsymbol{A}\right)^{-1}\boldsymbol{b} \tag{6}$$

where $\boldsymbol{A} = -(\boldsymbol{M}\boldsymbol{R}\boldsymbol{M}^T)^{-1}(\boldsymbol{M}\boldsymbol{L}\boldsymbol{M}^T)$ and $\boldsymbol{b} = (\boldsymbol{M}\boldsymbol{R}\boldsymbol{M}^T)^{-1}\boldsymbol{d}$.

To compute a Padé approximation, note that

$$\boldsymbol{Y}_{ij}(s) = \boldsymbol{c}^T\left(\boldsymbol{I} - s\boldsymbol{A}\right)^{-1}\boldsymbol{b} = \sum_{k=0}^{\infty} m_k s^k. \tag{7}$$

where

$$m_k = \boldsymbol{c}^T\boldsymbol{A}^k\boldsymbol{b} \tag{8}$$

is the k^{th} moment of the transfer function. A Padé approximation of q^{th} order is defined as the rational function

$$\boldsymbol{G}_q^P(s) = \frac{b_{q-1}s^{q-1} + \cdots + b_1 s + b_0}{a_q s^q + a_{q-1}s^{q-1} + \cdots + a_1 s + 1} \tag{9}$$

whose coefficients are selected to match the first $2q-1$ moments of the transfer function.

Padé approximates can be computed using direct evaluation of the moments, though the approach is ill-conditioned. Instead, Lanczos-style algorithms can be used [2]. An alternative approach, which robustly generates a somewhat different approximation, can be derived using the GMRES algorithm directly. The GMRES approach is better conditioned than direct evaluation of the moments because it generates a mutually orthogonal set of vectors which spans $\boldsymbol{A}^k\boldsymbol{b}, k = 0, \ldots, 2q-1$.

Note that the computation of $\boldsymbol{b}$ is inexpensive since $\boldsymbol{M}\boldsymbol{R}\boldsymbol{M}^T$ is sparse. Also, the dominant cost of each step of GMRES is a matrix-vector product, $\boldsymbol{A}\boldsymbol{x} = -(\boldsymbol{M}\boldsymbol{R}\boldsymbol{M}^T)^{-1}(\boldsymbol{M}\boldsymbol{L}\boldsymbol{M}^T)\boldsymbol{x}$, whose dense part, $(\boldsymbol{M}\boldsymbol{L}\boldsymbol{M}^T)\boldsymbol{x}$, can be computed with a hierarchical multipole-algorithm as in FastHenry.

After $2q$ steps of GMRES, one can show that for $k < 2q-1$,

$$\boldsymbol{A}^k\boldsymbol{b} = ||\boldsymbol{b}||\boldsymbol{A}^k\boldsymbol{V}_{2q}\boldsymbol{e}_1 = ||\boldsymbol{b}||\boldsymbol{V}_{2q}\boldsymbol{H}_{2q}^k\boldsymbol{e}_1 \tag{10}$$

where the columns of $\boldsymbol{V}_{2q} \in \mathbb{R}^{m\times 2q}$ are mutually orthogonal, $\boldsymbol{H}_{2q} \in \mathbb{R}^{2q\times 2q}$ is an upper Hessenberg matrix, and $\boldsymbol{e}_1 = [1\ 0\ldots 0]^T \in \mathbb{R}^{2q}$. With this relation, the moments (8) become

$$m_k = \boldsymbol{c}^T\boldsymbol{A}^k\boldsymbol{b} = ||\boldsymbol{b}||\boldsymbol{c}^T\boldsymbol{V}_{2q}\boldsymbol{H}_q^k\boldsymbol{e}_1 \tag{11}$$

and so the $2q^{th}$ order approximation to $\boldsymbol{Y}_{ij}$ can be written as

$$\boldsymbol{G}_{2q}^G(s) = ||\boldsymbol{b}||\boldsymbol{c}^T\boldsymbol{V}_{2q}\left(\boldsymbol{I} - s\boldsymbol{H}_{2q}\right)^{-1}\boldsymbol{e}_1. \tag{12}$$

Using the eigendecomposition $\boldsymbol{G}_{2q}^G = \boldsymbol{S}_{2q}\Lambda_{2q}\boldsymbol{S}_{2q}^{-1}$, the expression for the approximating rational function becomes

$$\boldsymbol{G}_{2q}^G(s) = ||\boldsymbol{b}||\sum_{k=0}^{2q}\frac{\mu_k\nu_k}{s - p_k} \tag{13}$$

where $\boldsymbol{\mu} = \boldsymbol{c}^T\boldsymbol{V}_{2q}\boldsymbol{S}_{2q}\Lambda_{2q}^{-1}$, $\boldsymbol{\nu} = \boldsymbol{S}_{2q}^{-1}\boldsymbol{e}_1$, and $\boldsymbol{p} = \Lambda_{2q}^{-1}$ are the poles of the approximation.

Note that the rational function $\boldsymbol{G}_{2q}^G(s)$ is *not* a Padé approximation, as it has $2q$ poles but only matches $2q-1$ moments. However, the method requires only $2q$ matrix-vector products, the same number of matrix-vector products required to compute a q^{th} order Padé approximate. And, a q^{th} order Padé approximation also matches $2q-1$ moments.

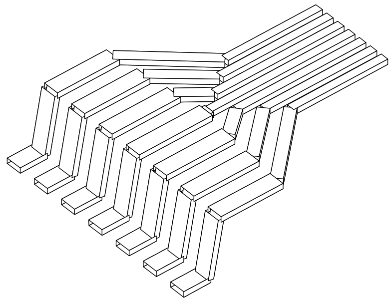

Figure 1: Seven pins of a cerquad pin package.

4 Preliminary Results

In the preceding section, we have described algorithms to compute Padé approximations of order q and GMRES-derived models of orders q and $2q$. In this section we compare the accuracy of these three approximations when used to obtain reduced-order models for the frequency-dependent admittance for a small set of package pins, as shown in Figure 1. To compute the resistance and inductance matrices with FASTHENRY, the pins were discretized into three filaments along their height and four along their length producing a system of size $m = 887$. This allows modeling of changes in resistance and inductance due to skin and proximity effects.

Figure 2 shows the relative error of the 8^{th} order Padé and the 8^{th} and 16^{th} order GMRES-derived approximations to the coupled admittance transfer function between pins 1 and 2. The figure indicates that the GMRES-derived approximations are more accurate than the Padé approximation. We note that from the figure we can infer that the 8^{th} order GMRES-derived approximation is more accurate than its equivalent Padé approximation of the same order, and it has half the computational cost, since only q GMRES iterations are performed.

Note also that if discrete frequency data were generated instead, and the approach described in [1] were used, generating 50 frequency points would require roughly 30 times the number of GMRES iterations needed by the direct state-space approach just described.

5 Conclusions and Acknowledgments

In this paper we described an accurate approach for using the iterative method in the FASTHENRY program to compute reduced-order models of frequency-dependent inductance matrices associated with complicated 3-D structures. The key advantage of this new method is that it is no more

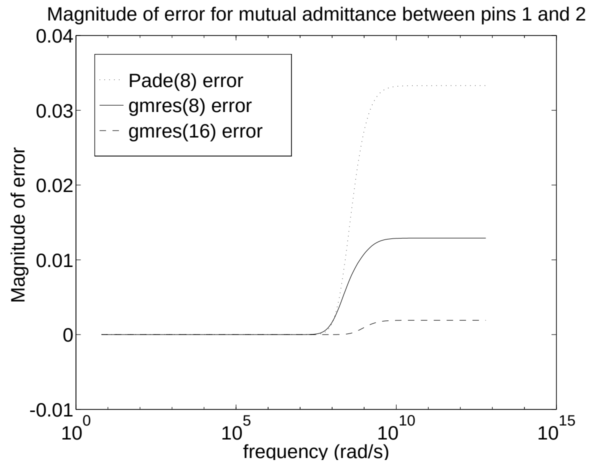

Figure 2: Relative errors for the approximations $G_8^P(s)$, $G_8^G(s)$ and $G_{16}^G(s)$ to the coupled admittance transfer function between pins 1 and 2.

expensive than computing the inductance matrix at a single frequency.

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