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# ALGORITHMS FOR EFFICIENT 3-D INTERCONNECT MODELING AND COUPLED CIRCUIT SIMULATION

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**Abstract** —

*The ever increasing speeds and shrinking feature sizes that are typical of state of the art integrated circuits designs have made coupling due to interconnect and packaging a very important, sometimes dominant, factor in system performance. The ability to efficiently perform coupled circuit-interconnect simulation before fabrication is essential in order to detect signal degradation due to delays or crosstalk. In this paper we review some algorithms currently used for generating low order models for efficient coupled circuit-interconnect simulation. We first discuss methods of generating models for three dimensional interconnect and packaging structures and then present state-of-the-art model order reduction techniques to reduce the order of the interconnect models.*

## I. INTRODUCTION

The dense interconnections and packaging structures used in compact electronic systems often produce electrical and magnetic interactions which interfere with system performance. With higher speeds, shrinking feature sizes and increasing complexity, the electrical and magnetic characteristics of such structures are becoming increasingly important factors in the behavior and performance of currently fabricated integrated circuits. Interconnect effects such as ringing, reflection, crosstalk, dispersion and attenuation may corrupt logic signals and prevent a circuit from functioning as specified. Such effects are difficult to simulate because they occur only as a result of an interaction between the field distribution in a complicated geometry of conductors, and the circuitry connected to those conductors.

For three dimensional structures small compared to a wavelength, electromagnetic interactions between conductors can be represented arbitrarily accurately using a densely coupled resistor, inductor, and capacitor (RLC) network [1]. Although it is possible to simulate coupled circuit-interconnect problems by including this densely coupled RLC network with the transistor models in a circuit simulator, this can be a very inefficient approach. In fact, to reasonably accurately model complicated interconnect and packaging these *RLC* models require a very large number of energy storage elements, and this will make subsequent circuit simulations using the models prohibitively expensive. Impulse response or recursive convolution techniques could be used, but more recently, model-order reduction schemes have been developed to reduce the number of these energy-storage elements *without* compromising the transient response properties of the model [2, 3, 4, 5].

In this paper we review state-of-the-art algorithms for accurate modeling of interconnect and packaging structures for coupled circuit-interconnect simulation. As an illustrative example we will describe a numerically robust and accurate approach for computing reduced-order models of magnetoquasistatic coupling in complicated 3-D structures. The approach is based on using the multipole-accelerated program FASTHENRY [6], combined with the Krylov-subspace based Arnoldi algorithm [7]. The techniques discussed are representative of currently available algorithms for extraction and modeling of 3D interconnect. For an example of modeling capacitance coupling effects see [8]. For a more recent formulation allowing simultaneous modeling of electrical and magnetic coupling see [9]. We begin, in section II, by describing the mesh-formulation approach of FASTHENRY. Then, in section III we discuss commonly used techniques for performing model order reduction on interconnect models and describe in some detail the Arnoldi-based model order reduction algorithm. Next, in section IV we review the relevant issues that are involved in including interconnect models into standard circuit simulators such as SPICE or SPECTRE. In section V we describe examples that illustrate the techniques described. Finally, in section VI, we present conclusions and acknowledgments.

## II. INTERCONNECT AND PACKAGE MODELING

In this section we describe some state-of-the art techniques available for generating models of three-dimensional interconnect amenable to model order reduction.

## Modeling 3D Magnetic Coupling

A wide range of integrated circuit and packaging design problems require accurate estimates of the coupling inductances of complicated three-dimensional structures. The frequencies of interest generally require magnetoquasistatic analysis, and a well known approach is to use the Partial Element Equivalent Circuit (PEEC) method [10]. This approach generates dense matrices which can be too expensive to solve by direct matrix factorization.

More recently a method has been proposed that accelerates the dense matrix solution allowing the modeling of complex packaging structures [6]. This method uses the standard PEEC method [10], but reformulates the discretized equations using mesh analysis. The dense system of equations is solved iteratively with a Krylov-subspace method known as GMRES [7], and the dense matrix product within each iteration is accelerated with the fast multipole method [11, 8]. The combination of these techniques has been implemented in a packaging analysis program, called FASTHENRY, whose computational complexity and memory requirements grow nearly linearly with the number of volume-elements required to discretize the conductors. A more recent alternative to the fast multipole method is the use of precorrected-FFT methods which at least for the capacitive coupling problem have been shown to have some advantages [12].

In FASTHENRY each conductor in a $t$-conductor system is approximated as piecewise-straight sections. To model skin and proximity effects, the volume of each straight section is then discretized into a collection of parallel thin filaments through which current is assumed to flow uniformly. By applying the Method of Moments to the magnetoquastatic equations, one can assign to each filament a resistance, inductance, and mutual inductance to every other filament in the problem.

A system of equations for the filament currents can be derived by assuming that the applied currents and voltages are sinusoidal, and that the system is in sinusoidal steady-state. Following the partial inductance approach in [13], the filament current phasors can be related to filament voltage phasors by a branch impedance matrix, $\boldsymbol{Z} = \boldsymbol{R} + j\omega\boldsymbol{L}$.

The planar graph interconnecting the filaments creates a circuit and combining this formulation with Kirchhoff's voltage law via mesh analysis, the following equation relating the vector of mesh currents and the source branch voltages can be obtained

$$\boldsymbol{M}\boldsymbol{Z}\boldsymbol{M}^T\boldsymbol{I}_m = \boldsymbol{V}_s. \tag{1}$$

where $\boldsymbol{M} \in \mathbb{R}^{m\times b}$ is the mesh matrix, $\boldsymbol{V}_s \in \mathbb{C}^m$ is the mostly zero vector of source branch voltages and $\boldsymbol{I}_m \in \mathbb{C}^m$ is the vector of mesh currents.

For a specific $\omega$, the complex admittance matrix which describes the external terminal behavior of a $t$-conductor system, denoted $\boldsymbol{Y}_t = \boldsymbol{Z}_t^{-1}$, can by derived from (1) by noting that $\boldsymbol{I}_t = \boldsymbol{Y}_t\boldsymbol{V}_t$ where $\boldsymbol{I}_t$ and $\boldsymbol{V}_t$ are the terminal source currents and voltages of the $t$-conductor system. These values are related to the mesh quantities by $\boldsymbol{I}_t = \boldsymbol{N}^T\boldsymbol{I}_m, \boldsymbol{V}_s = \boldsymbol{N}\boldsymbol{V}_t$, where $\boldsymbol{N} \in \mathbb{R}^{m\times t}$ is an easily constructed terminal incidence matrix. Hence, to compute the $i^{th}$ column of $\boldsymbol{Y}_t$, set $\boldsymbol{V}_{t_i} = 1$ and the rest of $\boldsymbol{V}_{t_i}$ to zero. Solve (1) with a $\boldsymbol{V}_s = \boldsymbol{N}\boldsymbol{V}_t$ and then extract the entries of $\boldsymbol{I}_m$ associated with the source branches via $\boldsymbol{I}_t = \boldsymbol{N}^T\boldsymbol{I}_m$. By choosing a carefully selected set of frequencies, the 3-D field solver can be repeatedly used to compute admitance matrices over a range of frequencies.

## Direct Model Generation

Given a frequency-dependent model described by a table of admitance values at a number of previously selected frequencies, a rational function fitting algorithm can be used to produce a model in transfer-function form [4]. This approach has been shown to produce accurate frequency-domain reduced-order models which are directly amenable to inclusion into a standard circuit simulator. Such techniques can in fact be used whether the frequency-dependent admitance data is obtained via measurement, an analytical model, or an electromagnetic tool [4]. However, in order to use frequency-domain fitting, it is necessary to use the field solver to compute impedance matrices at dozens of frequency points, and this is computationally expensive.

Another approach to coupling the above interconnect and package models with circuits is to include a sparse tableau version of (1) in a circuit simulator instead of solving for the terminal behavior [1]. This approach has the drawback that the size of the system in (1) can easily be very large if high-accuracy is desired. It is possible to derive a more efficient approach by exploiting the fact that 3-D field solvers typically use Krylov-subspace based iterative methods. These iterative methods can provide more than just a solution at a particular frequency, they can be used to directly construct reduced-order models [14]. To derive such a model consider forming a state-space representation of (1),

$$\begin{aligned} \frac{d}{dt}(\boldsymbol{M}\boldsymbol{L}\boldsymbol{M}^T)\boldsymbol{I}_m &= -(\boldsymbol{M}\boldsymbol{R}\boldsymbol{M}^T)\boldsymbol{I}_m + \boldsymbol{N}\boldsymbol{V}_t \\ \boldsymbol{I}_t &= \boldsymbol{N}^T\boldsymbol{I}_m. \end{aligned} \tag{2}$$

where $\boldsymbol{Z}$ has been expanded to $\boldsymbol{R} + j\omega\boldsymbol{L}$, and $j\omega$ replaced by $\frac{d}{dt}$. Direct inclusion of such a model in a simulator would still be too expensive given the size of system. However, model order reduction techniques as described in the next section can be applied to this formulation to derive a smaller approximation.

## III. MODEL ORDER REDUCTION

### Padé Approximations

Since the first papers on asymptotic waveform evaluation (AWE) [2], Padé-based reduced order models have become standard for improving coupled circuit-interconnect simulation efficiency. A Padé approximation of $q^{th}$ order is defined as a rational function whose coefficients are selected to match the first $2q-1$ moments of the transfer function of the system. For a SISO system given in state-space form

$$\begin{aligned} \boldsymbol{A}\,\dot{\boldsymbol{x}} &= \boldsymbol{x} + \boldsymbol{b}u \\ y &= \boldsymbol{c}^T\boldsymbol{x}. \end{aligned} \tag{3}$$

or in transfer function form $\boldsymbol{G}(s) = \boldsymbol{c}^T\,(\boldsymbol{I} - s\boldsymbol{A})^{-1}\,\boldsymbol{b} = \sum_{k=0}^{\infty} m_k s^k$, the moments are given as $m_k = \boldsymbol{c}^T\boldsymbol{A}^k\boldsymbol{b}$. Low order Padé approximates can be computed using direct evaluation of the moments, followed by a moment-matching procedure. In order to accurately compute higher order Padé approximates, it is necessary to use successive bi-orthogonalization combined with lookahead, as in the recent nonsymmetric Lanczos algorithms [14]. Although nonsymmetric Lanczos methods plus lookahead can be used to generate Padé approximates of arbitrarily high order, there is no guarantee that a given approximate will be stable. It is therefore essential to postprocess the Padé approximate before using it in a circuit simulation program.

### Arnoldi-based Approximations

An alternative approach, which robustly generates a somewhat different approximation, can be derived using an Arnoldi process as in the GMRES algorithm. The idea behind this approach is similar to that of [14], and is that of selecting an orthonormal basis for the Krylov subspace $\mathcal{K}_k(\boldsymbol{A}, \boldsymbol{b}) = span\{\boldsymbol{b}, \boldsymbol{A}\boldsymbol{b}, \boldsymbol{A}^2\boldsymbol{b}, \cdots, \boldsymbol{A}^{k-1}\boldsymbol{b}\}$. The orthogonality between the basis vectors makes the Arnoldi algorithm a better conditioned process than direct evaluation of the moments.

After $q$ steps, the Arnoldi algorithm returns a set of $q$ orthonormal vectors, as the columns of a matrix $\boldsymbol{V}_q \in \mathbb{R}^{n\times q}$, and a $q \times q$ upper Hessenberg (tridiagonal plus upper triangular) matrix $\boldsymbol{H}_q$. These two matrices are related by $\boldsymbol{V}_q^T\boldsymbol{A}\,\boldsymbol{V}_q = \boldsymbol{H}_q$. From this relation it can easily be shown that the moments of the transfer function can be written in terms of $\boldsymbol{H}_q$ and $\boldsymbol{V}_q$. Therefore the $q^{th}$ order Arnoldi-based approximation to $\boldsymbol{G}(s)$ can be written as

$$\boldsymbol{G}_q^A(s) = \|\boldsymbol{b}\|_2\,\boldsymbol{c}^T\,\boldsymbol{V}_q\,\left(\boldsymbol{I} - s\boldsymbol{H}_q\right)^{-1}\,\boldsymbol{e}_1 \tag{4}$$

corresponding to the state-space realization $\boldsymbol{A}_q = \boldsymbol{H}_q$, $\boldsymbol{b}_q = \boldsymbol{e}_1$, and $\boldsymbol{c}_q = \|\boldsymbol{b}\|_2\,\boldsymbol{V}_q^T\,\boldsymbol{c}$.

Block extensions to the Arnoldi approximation method, as well as the Lanczos method, can be derived, which allow for direct computation of reduced-order models for interconnect or packaging structures with multiple inputs and outputs from the general MIMO system equivalent to (3).

For certain classes of RC, RL or LC circuits it has been shown that congruence transforms, like the Arnoldi algorithm, can generate guaranteed stable and passive reduced-order models [15]. Recently a coordinate-transformed Arnoldi algorithm has been presented, that can generate arbitrarily accurate and guaranteed stable reduced-order models for RLC circuits. Efficient implementation of this algorithm requires simple modifications to the standard Arnoldi algorithm and its computational cost is roughly equivalent to that of PVL [5].

### Arnoldi Models for 3D Coupling

Application of the Arnoldi technique to the 3D modeling problem is straightforward from a state-space formulation of the problem. For instance, for the 3D coupling problem in packaging structures described in Section II.2, and from Eqn. (2) for the SISO case, the state-space formulation is readily obtained with $\boldsymbol{A} = -(\boldsymbol{MRM}^T)^{-1}(\boldsymbol{MLM}^T)$, $\boldsymbol{b} = (\boldsymbol{MRM}^T)^{-1}\boldsymbol{N}_j$ and $\boldsymbol{c} = \boldsymbol{N}_i$, where $\boldsymbol{N}_i$ indicates the $i^{th}$ column of $\boldsymbol{N}$.

Note that the computation of $\boldsymbol{b}$ is inexpensive since $\boldsymbol{MRM}^T$ is sparse. Also, because $L$ is dense, the dominant cost of each step of an Arnoldi process is a matrix-vector product, $\boldsymbol{Ax} = -(\boldsymbol{MRM}^T)^{-1}(\boldsymbol{MLM}^T)\boldsymbol{x}$. In practice, the matrix-vector cost dominates even when the dense part, $(\boldsymbol{MLM}^T)\boldsymbol{x}$, is rapidly computed with a hierarchical multipole-algorithm as in FASTHENRY.

A modified version of the standard Arnoldi algorithm which implements the coordinate-transformation described in [5] can easily be derived. This algorithm can be shown to efficiently generate arbitrarily accurate and guaranteed stable reduced-order models for this problem.

## IV. TIME-DOMAIN SIMULATION

In order to account for the interconnect or package coupling effects when verifying the correctness of a design, it is necessary to include the interconnect model in the verification tool. Usually this implies that the generated model must be amenable to inclusion in a standard circuit simulation tool such as SPICE [16] or SPECTRE.

If the coupling effects are being obtained through some experimental procedure or from repeated usage of some detailed analytical tool that generates frequency-dependent values, then the standard way to include this data into a simulator is via a convolution process at each timepoint. From the frequency-data, Laplace inversion or application of the inverse fast Fourier transform can be used to compute a corresponding impulse response which is then convolved with the excitation waveform. This approach is computationally demanding since at every simulator timepoint, the impulse response must be convolved with the entire computed excitation waveform. However, if some weak assumptions are made about the time-domain response, namely that it becomes "smooth" for large $t$, then the cost of computing the convolution integral can be significantly reduced [17].

If the frequency-domain data is approximated by a rational function then a recursive algorithm can be used which reduces the cost of computing the convolution integral at each timepoint to a constant factor [18].

A different approach can be applied if the interconnect model is in state-space form. In this case it is possible to directly include the model into the simulator, given that the model itself is simply a set of ordinary differential equations, similar to the circuit equations themselves. Therefore one can simply enlarge the list of variables that the simulator will solve for by adding to it the state vector. Given that the computational cost of circuit simulation grows superlinearly with the number of unknowns, it is important that the interconnect model be as small as possible.

Of course if the generated model is realizable then it is also possible to synthesize an RLC circuit from the state equations. In this case no modifications need to be introduced to the simulator and the model can be used directly.

## V. EXPERIMENTAL RESULTS

In the preceding sections, we described algorithms to compute reduced order models of interconnect and packaging structures using either Padé or Arnoldi-based algorithms. In this section the accuracy of these approximations is compared, first for a lumped-equivalent circuit for a three dimensional electromagnetic problem modeled via PEEC, and also when used to obtain reduced-order models for the frequency-dependent admittance for a small set of package pins. For this last example, the model generated for the packaging strcuture is used in a coupled circuit interconnect simulation that shows how crosstalk between the pins of the package can affect the performance and integrity of the signals.

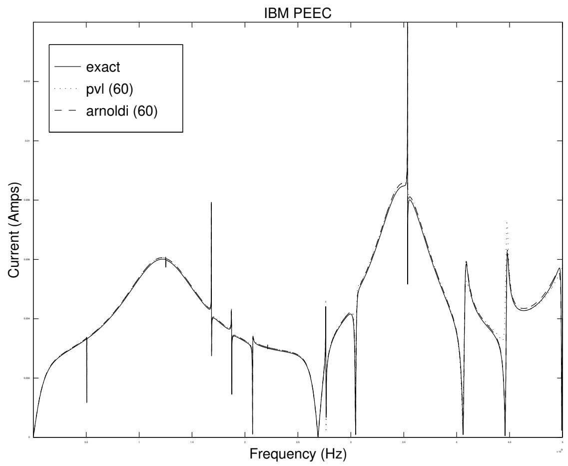


Figure 1: Circuit for 3-D problem modeled via PEEC. Shown in the figure are the exact solution, and $60^{\underline{th}}$ order PVL and stable Arnoldi approximations. Both are able to reproduce the transfer function with high accuracy.

### PEEC Example

This first example was introduced in [14]. The network is the lumped-element equivalent circuit for a three-dimensional problem modeled via PEEC. The circuit consists of 2100 capacitors, 172 inductors and 6990 inductive couplings, resulting in a $304 \times 304$ dense MNA matrix. In [14] it was shown that a $60^{\underline{th}}$ order approximation computed with PVL was able to reproduce the exact transfer function of the equivalent circuit. However, it was also reported, that some of the poles obtained with the PVL algorithm had positive real parts, albeit small. If the approximation is intended to be used within a circuit simulator, post-processing is required to eliminate such poles, which can be done if their residues are very small. The approximation obtained with a modified version of the Arnoldi algorithm described in [5], shown for comparison in Figure 1, can be seen to be of comparable accuracy, and is guaranteed stable. In fact the converged poles in this approximation all have nonpositive real parts. It can therefore be used in a circuit simulator unmodified.

### Package Example

Consider the small set of package pins. as shown in Figure 2. To compute the resistance and inductance matrices with FASTHENRY, the pins were discretized into three filaments along their height and four along their width producing a system of size $m = 887$. This allows modeling of changes in resistance and inductance due to skin and proximity effects. From the point of view of the terminals, the model to be produced has seven inputs and seven outputs.

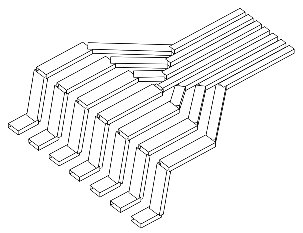

Figure 2: Seven pins of a cerquad pin package.

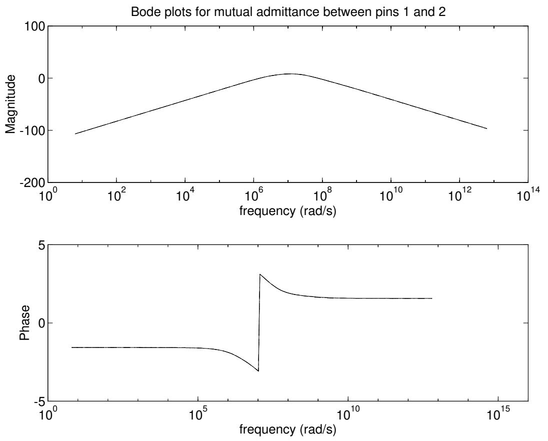


Figure 3: Bode plot for an Arnoldi approximation to the coupled admittance transfer function between pins 1 and 2. The curves are indistinguishable from the exact solutions which are also plotted.

Figure 3 shows the Bode plot of an $8^{th}$ order Arnoldi-derived approximations to the coupled admittance transfer function between pins 1 and 2. Also shown in the picture is the exact admittance transfer function obtained by eigendecomposition of the full system. As can be seen from the plot, the approximated model is virtually indistinguishable from the full transfer function.

For an example of how the generated models can be used to perform coupled circuit-interconnect simulation, consider the crosstalk between the pins in Fig. 2. Assume the five middle lines carry output signals from the chip and the two outer pins carry power and ground. The signals are driven and received with CMOS inverters. The drivers are capable of driving a large current to compensate for the impedance of the package pins. The inductance of the pins is modelled as described in the previous sections, and the capacitance is assumed to be 8pF. The interconnect on the end of a pin at the receiver side is modelled with a capacitance of 5pF. The overall configuration is illustrated in Fig. 4. A $0.1\mu F$ decoupling capacitor is connected between the driver's power and ground to minimize supply fluctuations.

The Arnoldi reduced-order model for each entry in the admittance matrix is incorporated into SPICE3 as a frequency-dependent voltage-controlled current source,

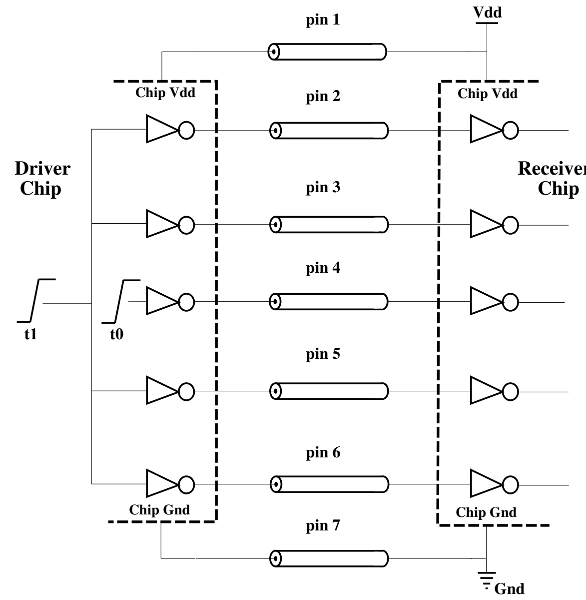


Figure 4: General configuration for the connection between received and driver chips. All the circuit elements inside the same chip share that chip's power and ground.

VCCS. As a sample time domain simulation, imagine that at time $t_0 = 4$ns the signal on pin 4 of Fig. 4 is to switch from high to low and pins $2, 3, 5$, and 6 are to switch from low to high but that due to delay on chip, pins $2, 3, 5$, and 6 switch at $t_1 = 5$ns. In this case, significant current will suddenly pass through the late pins while pin 4 is in transition. Due to crosstalk, this large transient of current has significant effects on the input of the receiver on pin 4, as shown in Fig. 5. Note that the input does not rise monotonically. Fig. 5 also shows that the bump in the waveform is carried through to the output of receiver, as a large glitch.

This example shows how reduced-order modeling techniques can be used to generate models for coupled circuit-interconnect simulation.

## VI. CONCLUSIONS

In this paper we reviewed some of the commonly used techniques for generating accurate models for interconnect and packaging coupling, that are amenable to efficient circuit level simulation. As an example of the capabilities and effectiveness of such techniques, an approach for computing reduced-order models of magnetoquasistatic coupling in complicated 3-D structures was reviewed. This approach shows how the combination of an appropriate formulation of the problem with the use of robust numerical techniques can lead to models that can be used to produce results within acceptable timeframes without compromising accuracy. Numerical examples were presented that show how ac-

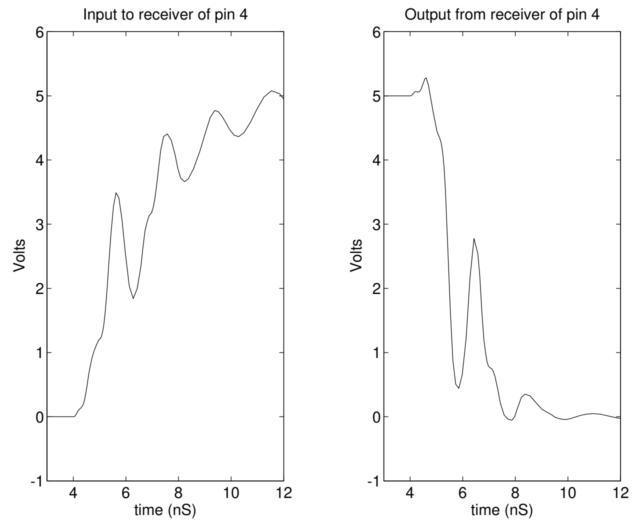


Figure 5: Results of the timing simulation of a receiver pin in the presence of changes on other adjacent pins. Pin 4's receiver input and output waveforms when four adjacent pins switch 1ns after pin 4.

curate reduced-order models can be generated and used in coupled circuit-interconnect simulation.

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